Amateur Radio: Basic & Advanced Exam Worked Examples

Draft Copy

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[This is a draft-copy of a study guide intended to aid those who wish to obtain their Basic or Advanced Amateur Radio Certification. These worked examples are specifically tailored to those individuals who do not have a strong background in math. Questions in this study guide are either taken directly from the basic and advanced question banks (RIC-7 and RIC-8 respectively) or are inspired in-part by the question banks.]

Preface:

The intention of this study guide is to offer step-by-step explanations for most of the commonly asked equations during Basic and Advanced Amateur Radio Certification examinations. This guide is specifically intended for individuals who do not have a strong background in formal mathematics, or have been far removed from their formal mathematical training.

Each step is broken down to (in general) only one mathematical operation at a time. In cases where it would be simply too elementary to show an operation, it is assumed that the reader has some basic math abilities. As an example, it is imperative to understand that $\frac{1}{4}$ functionally means 1 divided by 4.

Despite the assumption of some basic mathematical awareness, a brief review of some algebraic concepts and an introduction to stylistic formatting is presented prior to any worked examples. In some cases, even basic operations are explained or demonstrated with proofs due to their importance or complexity.

All questions in this study guide are provided with the assumption that the reader will only read the particular question of interest, and so you may find a great deal of repetition in the algebraic explanations. It is hoped that the degree of repetition will help reinforce the discussed operations for future use.

This study guide makes no attempt to teach all of the concepts required to pass the Basic and Advanced examinations. There is simply no substitute for good quality instruction as provided by clubs.

As this is a work-in-progress, please do not distribute this document in any form without my prior written consent (by email). This document has not been proof-read and is likely rife with literary errors. Every effort has been made to ensure the accuracy the math, and in fact, the answers given have been confirmed by the answers in the question banks. Having said that, there may still be some errors to be found.

In exchange for access to this draft document, all I ask is that you forward any : errors found (mathematical, stylistic, grammatical, spelling, etc.), points which require further discussion, unclear explanations, or any questions you wish to see covered (that are not already covered using other "given values"). I can be reached at: james@ve3bux.com

Thanks for your investment of time in this project and good luck with your studies,

James Buck

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Math Primer:

$$(\mathrm{A})(\mathrm{B}) = (\mathrm{A})\mathrm{x}(\mathrm{B})$$

Shown here is a basic convention for multiplication, in the worked examples I will be using $(\mathrm{A})(\mathrm{B})$ notation since it is more compact and organizes values nicely.

$$(\mathrm{C})^{-2}(\mathrm{C})^{3} = \mathrm{C}^{(-2+3)} = \mathrm{C}^{1}$$

$$\frac{(\mathrm{C})^{7}}{(\mathrm{C})^{4}} = \mathrm{C}^{(7-4)} = \mathrm{C}^{3}$$

For exponent math, when you multiply two exponents with the same base (in this case, 'C') you add the exponents. When you divide two exponents with the same base, you subtract the exponents.

$$(\mathrm{D}^{-4})^{5} = \mathrm{D}^{(-4)(5)} = \mathrm{D}^{-20}$$

If you raise an exponential value to the power of another exponential, you multiply the exponents.

$$\frac{1}{(\mathrm{EF})} = (\mathrm{EF})^{-1}$$

A reciprocal value, or an inverse, is any expression where the numerator is 1. Technically, $\frac{1}{2}$ is both a fraction and a reciprocal value of 2.

$$\sqrt{(\mathrm{GH})} = \sqrt[2]{(\mathrm{GH})} = (\mathrm{GH})^{\frac{1}{2}}$$

Square roots (also known as "radicals") can be expressed by an exponential value, but the exponent is a fraction.

$$(\mathrm{Jx}10^{9})(\mathrm{Kx}10^{-3}) = (\mathrm{J})(\mathrm{K})(10^{9})(10^{-3})$$
$$(\mathrm{Jx}10^{9})(\mathrm{Kx}10^{-3}) = (\mathrm{J})(\mathrm{K})(10^{6})$$

When working in scientific notation, you are able to separate the coefficients (J and K in this case) from their exponential values. This allows you to perform basic arithmetic without the need of an advanced calculator. Notice how the exponentials have been solved, this allows us to intelligently keep track of the decimal points, avoiding so called magnitude errors.

$$(\mathrm{Lx}10^{\mathrm{m}}) = \mathrm{LEm}$$
$$(5.0 \mathrm{~x~} 10^{2}) = 5\mathrm{E}2$$

When using a computer or a calculator, it is often more convenient to express scientific notation using the upper case "E" which represents $\mathrm{x}10$. Do not confuse upper case 'E' with the lower case 'e^{x}'. I highly recommend that if your calculator has the "E" or "EE" function, become familiar with its use! Test it by entering: 2E3 and you should get the result: 2000 when you hit enter/calc

Q1: Unit Conversion: mA to A
If an ammeter marked in amperes is used to measure 3500 milliamperes (3500mA), what reading will the ammeter show?

A1a: We are given a value in non-base units (mA) and asked what a meter that uses base units (A) would show. This means we need to convert from milliamperes (mA) to amperes (the base unit).

given: $I = 3500mA$

$$I = \frac{3500mA}{1} \text{x} \frac{1A}{1000mA} \quad (1)$$

$$I = \frac{(3500)(1A)}{(1000)} \quad (2)$$

$$I = \frac{35A}{10} \quad (3)$$

$$\boldsymbol{I = 3.5A}$$

Notes:

1. Multiply the given value by the fraction which has the base unit in the numerator, cross-cancelling any similar units
2. Simplify the equation and multiply by the unit that came over from the fraction multiplication.
3. Divide the 35A by 10 to obtain 3.5A

A1b: This time we will use scientific notation to convert from mA to A

$$\text{I} = 3\underbrace{500}\,mA = 3.5\text{x}10^3 mA \quad (1)$$

$$\text{I} = \frac{(3.5)(10^3 mA)}{1} \text{x} \frac{1A}{(10^3 mA)} \quad (2)$$

$$\mathbf{I = 3.5}\boldsymbol{A}$$

Notes:

1. Convert 3500mA to scientific notation by shifting the decimal place to the left 3 times, thus the exponential value becomes10^{+3}. When we convert a value to scientific notation, it is possible to separate the coefficient (x.x) from the exponential value (10^y). In this case, $3.5\text{x}10^3\text{mA}$ can be shown as $(3.5)(10^3\text{mA})$ which is useful for simplifying the math later.
2. Multiply our unit by the fraction which gives us A in the numerator and mA in the denominator. Cancel out like terms, leaving us with the coefficient and the base unit from the numerator.

To set the conversion fractions up for yourself, recall that milli means thousandths (as in a fraction of) and it is: $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$.

So 1 milliamp is $\frac{1}{1000}$ of an amp which is exactly the same as saying $\frac{1\text{A}}{1000\text{mA}}$ but we used scientific notation to make it quicker and more obvious.

Q2: Ohm's Law: Resistance

Determine the value of an unknown resistor (R1) in an ideal series circuit (wires have no resistance, battery has no internal resistance) where the current that flows is measured to be 3mA at 6V.

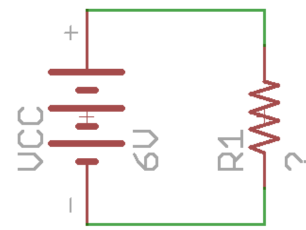


Figure 1: Series resistor circuit

A2: Since we know E and I, we rearrange Ohm's law to solve for R:

$$\mathbf{E = IR}$$

given: $\text{E} = 6\text{V}$ $\quad\text{I} = 3\text{mA} = 0.\underbrace{003}\,\text{A} = (3\text{x}10^{-3}\text{A})$

$$\frac{\text{E}}{\text{I}} = \frac{\text{IR}}{\text{I}} \quad (1)$$

$$\frac{\text{E}}{\text{I}} = \text{R} \quad (2)$$

$$\text{R} = \frac{6\text{V}}{(3\text{x}10^{-3}\text{A})} \quad (3)$$

$$\text{R} = \frac{6\text{V}}{3\text{A}}\text{x}\frac{1}{10^{-3}} \quad (4)$$

$$\text{R} = \frac{6\text{V}}{3\text{A}}(10^{-3})^{-1} \quad (5)$$

$$\text{R} = 2\text{ x }10^{3}\Omega \quad (6)$$

$$\text{R} = \frac{2\text{x}10^{3}\Omega}{1}\text{x}\frac{1\text{k}\Omega}{10^{3}\Omega} \quad (7)$$

$$\mathbf{R = 2k\Omega}$$

Notes:

1. Divide both sides of Ohm's law by I to begin isolating the variable which we want to find (R) and then cancel the current (I) on the right hand side of the equation since $\frac{\text{I}}{\text{I}} = 1$
2. With Ohm's law rearranged to isolate R, insert the known values to solve the equation
3. Separate the current coefficient from its exponential value
4. Notice how the exponential is now a reciprocal? We can move it to the numerator according to the example in the math primer! $\frac{1}{10^{-3}} = (10^{-3})^{-1}$
5. Divide $\frac{6\text{V}}{3\text{A}}$ and perform the exponent math: $(10^{-3})^{-1} = 10^{(-3)(-1)} = 10^{3}$. We get Ω from $\frac{\text{V}}{\text{A}}$
6. We could technically stop here since we found the answer to be 2000Ω, however since there is $1\text{k}\Omega$ per $10^{3}\Omega$, multiply $(2\text{ x }10^{3}\Omega)$ by $\frac{1\text{k}\Omega}{10^{3}\Omega}$ to express the value in kΩ which is more appropriate. Then cross-cancel like terms to obtain a simplified result
7. Multiply the numerators to obtain the final value in kΩ

On a calculator, this exercise is much more straight-forward and has fewer steps as you can simply divide E by I so long as they are in their base SI units (ie. not mA!) thus $R = \frac{E}{I} = \frac{6V}{0.003A}$

Q3: Ohm's Law: Current

Assuming the use of ideal wires, and an ideal battery, what is the individual current in each resistor in the following circuit?

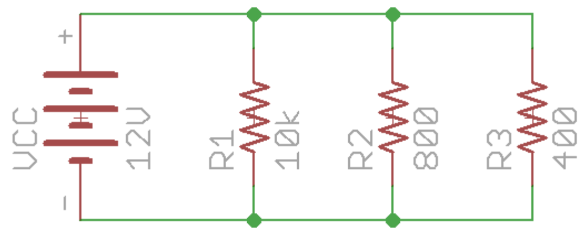


Figure 2: Parallel resistor circuit

A3: Remembering that in a parallel circuit, all points have the same voltage, we can easily determine the amount of current which flows through each resistor individually.

$$\boldsymbol{E = IR}$$

given: $E = 12V$

$R_1 = 10k\Omega = 1\underbrace{0000}\Omega = 1.0x10^4\Omega$

$R_2 = 800\Omega \quad R_3 = 400\Omega$

$$I_{R_1} = \frac{E}{R_1} \tag{1}$$

$$I_{R_1} = \frac{12V}{(1.0x10^4\Omega)} \tag{2}$$

$$I_{R_1} = (1.2)(10^1V)(10^{-4}\Omega^{-1}) \tag{3}$$

$$I_{R_1} = \frac{1.2x10^{-3}A}{1} x \frac{10^3mA}{1A} \tag{4}$$

$$\mathbf{I_{R_1} = 1.2mA}$$

Notes:

1. Rearrange Ohm's law to solve for I and substitute the values in the equation
2. To simplify the expression and do the arithmetic in our heads, we can move the denominator to the numerator by taking the inverse ie. $(1.0x10^4\Omega)^{-1} = (1.0)^{-1}(10^4)^{-1}$ which reduces to $(1.0)^{-1} = 1$ and $(10^4)^{-1} = 10^{(4)(-1)} = 10^{-4}$ thus $(1.0)^{-1}(10^{-4}) = 1(10^{-4}) = 10^{-4}$
3. Expressing 12V in scientific notation we then multiply the exponential values as: $(10^1)(10^{-4}) = 10^{(1-4)} = 10^{-3}$ and when we multiply $(V)(\Omega^{-1})$ it is the same as saying $\frac{V}{\Omega}$ which is Amps!
4. Knowing that there are 10^3mA per A, we can multiply by a conversion fraction to convert from A to mA. Notice that the exponent math cancels out as $(10^{-3})(10^3) = 10^{(3-3)} = 10^0 = 1$

To find the other currents, simply repeat the above process for R_2 and R_3. The values are given below:

$I_{R_2} = 15mA$ and $I_{R_3} = 30mA$

Q4: Ohm's Law: Current (Parallel resistor circuit)

Once again, using ideal wires, and an ideal battery, what is the total current in the following parallel-resistor circuit?

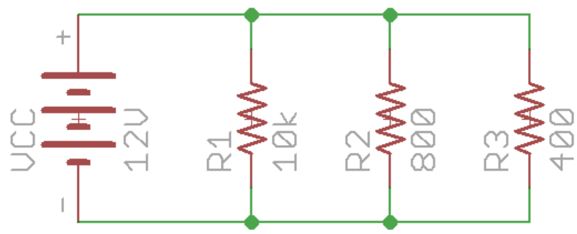


Figure 3: Parallel resistor circuit

A4a: The long method. Recall that in a parallel circuit, all points have the same voltage, but they share the total current. So according to Ohms law, the total current of this circuit would be found as:

$$\boldsymbol{I_{total} = \frac{E}{R_{total}}}$$

given: $E = 12V$

$R_1 = 10k\Omega = 1\underbrace{0000}\Omega = 1.0x10^4\Omega$

$R_2 = 800\Omega \qquad R_3 = 400\Omega$

$$I_{total} = I_{R_1} + I_{R_2} + I_{R_3} \quad (1)$$

$$I_{total} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} \quad (2)$$

$$I_{total} = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad (3)$$

$$I_{total} = E\left(\frac{1}{1.0x10^4\Omega} + \frac{1}{800\Omega} + \frac{1}{400\Omega}\right) \quad (4)$$

$$I_{total} = E((1.0x10^4\Omega)^{-1} + (800\Omega)^{-1} + (400\Omega)^{-1}) \quad (5)$$

Notes:

1. Knowing that in a parallel circuit the voltage is the same everywhere and that the current adds up, we know that we need to add the individual currents for each resistor.
2. Substitute $\frac{E}{R}$ into the I_{total} equation for each resistor and since the voltage does not change, and it is a common term, move it out to the front of the equation. $\left(\frac{a}{b} + \frac{a}{c} + \frac{a}{d}\right) = a\left(\frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right)$
3. Plug the variables into the equation.
4. If you do not have a scientific calculator, perform the arithmetic now. Otherwise, rearrange the fractional expression to an inverse expression as it is easier to use a scientific calculator this way.
5. Perform the arithmetic to solve the total resistance. Be sure you use the ( ) as suggested otherwise you risk introducing order-of-operations errors. Also, notice the Ω^{-1}

..continued..

$$I_{total} = (12V)(3.85\Omega^{-1})(10^{-3}) \quad (6)$$

$$I_{total} = (46.2A)(10^{-3}) \quad (7)$$

$$I_{total} = (4.62A)(10^{1})(10^{-3}) \quad (8)$$

$$I_{total} = \frac{(4.62A)(10^{-2})}{1} x \frac{10^{3}mA}{1A} \quad (9)$$

$$I_{total} = 4.62x10^{1}mA \quad (10)$$

$$\mathbf{I_{total} = 46mA}$$

Notes:

6. Insert the voltage and multiply the coefficients, finding that $Vx\ \Omega^{-1} = A$ (see Appendix)
7. Express the 46.2A as $4.62x10^{1}A$ and then group the exponentials
8. Perform the exponent math as follows: $(10^{1})(10^{-3}) = 10^{(1-3)} = 10^{-2}$
9. Since the value $4.62x10^{-2}A$ is awkward, lets convert it to mA by multiplying by a conversion fraction and then cancel any like terms (A). Notice the exponent math once again: $(10^{-2})(10^{3}) = 10^{(-2+3)} = 10^{1}$
10. Multiply the coefficient by its exponential to properly express the total current in mA

Since our voltage value carries the least accuracy (only 2 digits) we are justified in providing our answer as 46mA by rounding. This is quite an exhaustive exercise to do via the long method.

Let's now look at an alternate method of solving this. As you could imagine, the long method is particularly inefficient when dealing with many resistors. Recall from the discussion that we started the derivation of the basis of calculating the total resistance of a sequence of parallel resistors?

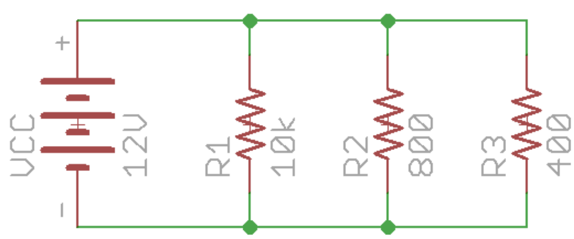


Figure 4: Parallel resistor circuit

The circuit shown below (figure 4) is equivalent to the one shown in figure 3 provided: R4 is said to have the total parallel resistance of all three resistors (R1, R2, and R3).

$$\mathbf{R_{total}} = \frac{\mathbf{1}}{\left(\frac{\mathbf{1}}{\mathbf{R_1}} + \frac{\mathbf{1}}{\mathbf{R_2}} .. + \frac{\mathbf{1}}{\mathbf{R_n}}\right)}$$

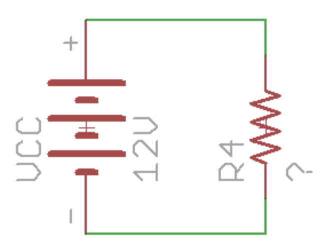


Figure 5: Equivalent series circuit

This is the equation which is used to determine the total resistance of a group of parallel resistors. If we were really keen, we could derive this formula from the worked example above using equal value resistors!

A4b: The quick method. Using Thévenin's theorem, we can simplify the three parallel resistors to a single equivalent resistor value, and then solve the circuit using Ohm's law. The diagram below shows the equivalent circuit of the original, however we have replaced the three parallel resistors with R_4, the value for which we will calculate as $(R_{parallel}) = (R_{total})$.

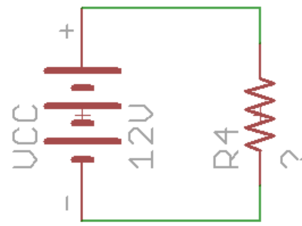


Figure 6: Unknown Resistance

$$\boldsymbol{R_{parallel}} = \frac{\mathbf{1}}{\left(\frac{1}{R_1}+\frac{1}{R_2}..+\frac{1}{R_n}\right)} = \left(\frac{1}{R_1}+\frac{1}{R_2}..+\frac{1}{R_n}\right)^{-1}$$

$$R_{total} = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1} \quad (1)$$

$$R_{total} = \left(\frac{1}{1.0x10^4\Omega}+\frac{1}{800\Omega}+\frac{1}{400\Omega}\right)^{-1} \quad (2)$$

$$R_{total} = [(1.0x10^4\Omega)^{-1}+(800\Omega)^{-1}+(400\Omega)^{-1}]^{-1} \quad (3)$$

$$R_{total} = 259\Omega \quad (4)$$

$$I_{total} = \frac{E}{R_{total}} = \frac{12V}{259\Omega} = 4.63x10^{-2}A \quad (5)$$

$$I_{total} = \frac{4.63x10^{-2}A}{1} x \frac{10^3mA}{1A} \quad (6)$$

$$I_{total} = 4.63x10^1mA \quad (7)$$

$$\mathbf{I_{total} = 46mA}$$

Notes:

1. Using the general formula for finding the total resistance for parallel resistors, substitute the known values in to the equation to determine the total resistance (or value of R_4).
2. Rearrange the equation into a scientific calculator-friendly format. If you do not have a scientific calculator, use the original fractional format to solve the total resistance.
3. Perform the arithmetic to find the final value. On a scientific calculator, the input would look similar to: $((1E^4)^{-1} + (800)^{-1} + (400)^{-1})^{-1}$ - be sure to use all the brackets () !!

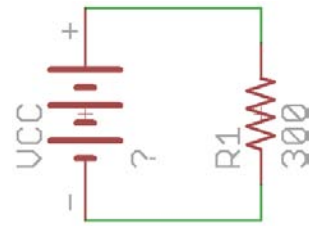
The equivalent circuit technique may be used for other circuits, just be sure you use the appropriate formula for each!

$R_{parallel}$; C_{series}; $L_{parallel}$ all use the general $\square_{total} = \frac{1}{\left(\frac{1}{\square_1}+\frac{1}{\square_2}..+\frac{1}{\square_n}\right)}$ form and were □ represents R,L,C

R_{series}; $C_{parallel}$; L_{series} all use the $\square_{total} = (\square_1 + \square_2..+\square_n)$ general form.

Q5: Ohm's Law: Voltage

What is the voltage of a circuit which draws a current of $50mA$ and has a total resistance of 300Ω?



A5a: Since we know what the current and resistance of the circuit are, we can use Ohm's law to find out what voltage is being applied.

$$\boldsymbol{E = IR}$$

given: $I = 50mA = 0.050A$
$R = 300\Omega$

$$E = (0.050A)(300\Omega) \qquad (1)$$

$$\boldsymbol{E = 15V}$$

Notes:

1. Using Ohm's law in its most basic form, plug the given values into the equation. Just be sure that you are working in base units!

A5b: This time, let's use scientific notation to solve this problem.

$$\boldsymbol{E = IR}$$

given: $\mathrm{I}_1 = 50mA = 0.\underbrace{05}0A = 5.0\mathrm{x}10^{-2}A$
$\mathrm{R}_1 = 300\Omega = 3.0x10^2\Omega$

$$E = (5.0x10^{-2}A)(3.0x10^2\Omega) \qquad (1)$$

$$E = (5.0)(3.0)(10^{-2}A)(10^2\Omega) \qquad (2)$$

$$\boldsymbol{E = 15\mathrm{V}}$$

Notes:

1. Input the given values into the equation in scientific notation
2. Group coefficients and exponential values and then perform the multiplication. For the exponents, the math goes as follows: $(10^{-2})(10^2) = 10^{(-2+2)} = 10^0 = 1$

Notice that by using scientific notation, we do not need a calculator to do the basic math. Solving the problem was as simple as multiplying 5 by 3 and cancelling the exponents.

Q6: Ohm's Law: Ratios

A meter has a full-scale deflection of 40µA and an internal resistance of 96Ω. You want the meter to read 0 to 1mA. The value of the shunt resistor to be used is:

A6: To solve this problem, we can take advantage of Ohm's law. Using the first set of given values, solve for voltage. With the value of voltage #1, use that value to find the missing resistance for the second equation.

$$\boldsymbol{E = IR}$$

given: $I_1 = 40\mu A = 4.0x10^{-5}A$
$R_1 = 96\Omega$
$I_2 = 1mA = 1.0x10^{-3}A$

$$V_1 = V_2 \quad (1)$$

$$(I_1)(R_1) = (I_2)(R_2) \quad (2)$$

$$R_2 = \frac{(I_1)(R_1)}{I_2} \quad (3)$$

$$R_2 = \frac{(4.0x10^{-5}\cancel{A})(96\Omega)}{1.0x10^{-3}\cancel{A}} \quad (4)$$

$$R_2 = \frac{(4.0)(96\Omega)(10^{-5})}{(1.0)(10^{-3})} \quad (5)$$

$$R_2 = \frac{(384\Omega)(10^{-5})}{(10^{-3})} \quad (6)$$

$$R_2 = (384\Omega)(10^{-2}) \quad (7)$$

$$\mathbf{R_2 = 3.8\Omega}$$

Notes:

1. Because we are going to modify the original meter, we must take into consideration the previous values. We do this by using Ohm's law. Since we can calculate the missing voltage, we will use voltage to find the new (missing) resistance value. Clever huh?
2. Since the voltages will be equal to each other, substitute Ohm's law into the equation and then rearrange the equation to isolate R_2 which is what we are looking for
3. Plug the known values into the reduced equation and cancel similar units.
4. Group coefficients and exponents and cancel like units (A)
5. Multiply the coefficients in the numerator at this time. Notice that the (1.0) in the denominator disappears? This is because $\frac{(4.0)(96\Omega)}{1} = (4.0)(96\Omega)$
6. Lets handle the exponents as follows: $\frac{(10^{-5})}{(10^{-3})} = 10^{(-5)-(-3)} = 10^{-2}$
7. Now multiply out the coefficient and the exponent (recall 10^{-2} effectively means move the decimal place 2 spots to the left). Also round the answer to one decimal place.

Q7: Power: Current and Resistance
What is the power rating of a light which draws a current of $2A$ and has a resistance of 25Ω?

A7: From the information given, we know that we are being asked to calculate the current in a circuit and must use Ohm's law. Since we are given the power (in watts) and the voltage, we can use the power equation which has voltage and current to solve the problem.

$$\boldsymbol{P = I^2R}$$

given: $\mathrm{I} = 2\mathrm{A}$
$R = 25\Omega$

$$\mathrm{P} = (2\mathrm{A})^2(25\Omega) \quad (1)$$

$$\mathrm{P} = (4\mathrm{A}^2)(25\Omega) \quad (2)$$

$$\boldsymbol{P = 100\mathrm{W}}$$

Notes:

1. Use Ohm's power law for current and resistance. Plug the given values into the equation and solve the $(2\mathrm{A})^2 = (4\mathrm{A}^2)$
2. Multiply the values out, noticing that $(\mathrm{A}^2)(\Omega) = \mathrm{W}$

Q8: Power: Finding Current using Voltage
A 12V light bulb is rated at a power of 60 watts at its rated voltage. How much current is being drawn from the voltage source to power the bulb?

A8: From the information given, we know that we are being asked to calculate the current in a circuit and must use Ohm's law. Since we are given the power (in watts) and the voltage, we can use the power equation which has voltage and current to solve the problem.

$$\boldsymbol{P = EI}$$

given: $\mathrm{E} = 12\mathrm{V}$
$\mathrm{P} = 60\mathrm{W}$

$$\mathrm{I} = \frac{\mathrm{P}}{\mathrm{E}} \quad (1)$$

$$\mathrm{I} = \frac{60\mathrm{W}}{12\mathrm{V}} \quad (2)$$

$$\mathbf{I = 5A}$$

Notes:

1. Rearrange the Ohm's power law and rearrange it to solve for current. Plug the given values into the equation to solve for the unknown current
2. Divide 60W by 12Vto solve. Recall that the answer is in amps!

Q9: Power: Finding Voltage using Resistance
A 5Ω resistor dissipates 45W of heat. What is the voltage that is passing through the resistor?

A9: Once again, we are being asked to solve a power question which requires rearranging Ohm's law to solve for voltage. This time we will use the power equation which has voltage and resistance as variables.

$$\boldsymbol{P = \frac{E^2}{R}}$$

given: $\mathrm{P = 45W}$
$\mathrm{R = 5\Omega}$

$$\mathrm{PR = \frac{E^2R}{R}} \quad (1)$$

$$\mathrm{PR = E^2} \quad (2)$$

$$\mathrm{\sqrt{E^2} = \sqrt{PR}} \quad (3)$$

$$\mathrm{E = \sqrt{PR}} \quad (4)$$

$$\mathrm{E = \sqrt{(45W)(5\Omega)}} \quad (5)$$

$$\mathrm{E = \sqrt{225V^2}} \quad (6)$$

$$\mathbf{E = 15V}$$

Notes:

1. To begin isolating E, multiplying both sides by R and then cancel where possible
2. Now that we have $\mathrm{E^2}$ by itself, we further simplify the equation by taking the square root of both sides of the equation
3. Recall that $\mathrm{\sqrt{E^2} = (E^2)^{\frac{1}{2}} = E^{\left(\frac{2}{1}\right)\left(\frac{1}{2}\right)} = E^{\frac{2}{2}} = E^1 = E}$
4. Input the variables into the equation
5. Solve $\mathrm{(45W)(5\Omega)}$ and notice the unit becomes $\mathrm{V^2}$
6. Take the square root of PxR to obtain our result

Q10: Power: Peak Envelope Transmitter Output
What is the output PEP from a transmitter if an oscilloscope measures 200 volts peak-to-peak across a 50Ω dummy load connected to the transmitter output?

A10a: Quick and dirty: PEP is Peak-Envelope-Power and it is essentially a measure of the average power in an alternating current (in our case, RF). All we need to do is make sure that we are working in single-peak RMS volts first, then we apply Ohms power law using E and R.

$$\mathbf{E_{peak} = \frac{E_{peak-to-peak}}{2}}$$

$$\mathbf{E_{rms} = (E_{peak})\left(0.707\frac{V_{rms}}{V_p}\right)}$$

$$\mathbf{P_{pep} = \frac{{E_{rms}}^2}{R}}$$

given: $E_{peak-to-peak} = 200V_{pp}$
$R = 50\Omega$

$$E_{peak} = \frac{200V_{pp}}{2} \quad (1)$$

$$E_{peak} = 100V_p$$

$$E_{rms} = (100)(0.707)\left(\frac{V_p}{1}\right)\left(\frac{V_{rms}}{V_p}\right) \quad (2)$$

$$E_{rms} = 70.7V_{rms}$$

$$P_{pep} = \frac{(70.7V)^2}{50\Omega} \quad (3)$$

$$\mathbf{P_{pep} = 99.9W \doteq 100W\ (after\ rounding)}$$

Notes:

1. Find the single-peak voltage by dividing the peak-to-peak value by 2. The peak-to-peak value is commonly measured using an oscilloscope since it is easiest to measure two peak-tops as opposed to trying to find the middle (zero value) of a sine-wave.
2. Convert the single-peak voltage into the average, or root-mean-square (RMS) value. Notice that the voltage units have been separated out, this is to formally show how the units change by cross-cancellation.
3. Simply plug the RMS voltage we have calculated along with the dummy-load resistance into the equation and then solve

This is the quick-and-dirty method of solving the question. Its admittedly simple, but not the most academic method of solving such a problem. Recall that we simply memorize the $0.707\left(\frac{V_{rms}}{V_p}\right)$. Well, technically the RMS value is found as $\frac{1}{\sqrt{2}}$ so let's try using that instead! Oh, and calculators are optional!

A10b: Ok, so we know that RMS $= \frac{1}{\sqrt{2}}$ and that the value is an irrational number which never ends, why not try using the "true" RMS value in an equation instead of the "constant" 0.707? We will also combine all the equations into a single formula, then input the variables and solve it in one shot.

$$\mathbf{E_{peak}} = \frac{\mathbf{E_{peak-to-peak}}}{\mathbf{2}}$$

$$\mathbf{E_{rms}} = (\mathbf{E_{peak}})\left(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right)\left(\frac{\mathbf{V_{rms}}}{\mathbf{V_{peak}}}\right)$$

$$\mathbf{P_{pep}} = \frac{\mathbf{E_{rms}}^2}{\mathbf{R}}$$

$$E_{rms} = \left(\frac{E_{peak-to-peak}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\cancel{V_{peak}}}{1}\right)\left(\frac{V_{rms}}{\cancel{V_{peak}}}\right) \quad (1)$$

$$E_{rms} = \left(\frac{E_{peak-to-peak}}{2\sqrt{2}}\right)(V_{rms}) \quad (2)$$

$$P_{pep} = \frac{\left((E_{peak-to-peak})\left(\frac{1}{\sqrt{8}}\right)(V_{rms})\right)^2}{R} \quad (3)$$

$$P_{pep} = \frac{(E_{peak-to-peak})^2(V_{rms})^2\left(\frac{1}{8}\right)}{R} \quad (4)$$

$$\mathbf{P_{pep}} = \frac{(\mathbf{E_{peak-to-peak}})^2(\mathbf{V_{rms}})^2}{\mathbf{8R}} \quad (5)$$

Notes:

1. We jump right in by substituting the equation for E_{peak} directly into the E_{rms} equation and cross-cancel like units now
2. Multiply the two fractions. As a guide, $\left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$, then substitute the E_{rms} equation into P_{pep} equation. By the relationship that $(\sqrt{a})(\sqrt{b}) = \sqrt{ab}$ we can say $(\sqrt{a^2})(\sqrt{b}) = \sqrt{a^2b}$ which we will use to convert $2\sqrt{2}$ into $\sqrt{(2)^2(2)}$ which reduces to: $\sqrt{(4)(2)} = \sqrt{8}$
3. The equation looks kind of ugly, so simplify it. Note that $(\sqrt{8})^2 = 8$ since $(\sqrt{a})^2$ effectively means: $\left(a^{\frac{1}{2}}\right)^2 = a^{\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)} = (a)^{\frac{2}{2}} = a^1 = a$
4. Notice the awkward $\frac{\left(\frac{1}{8}\right)}{R}$, which we can fix! It simplifies to $\frac{\left(\frac{1}{8}\right)}{\left(\frac{R}{1}\right)} = \left(\frac{1}{8}\right)\left(\frac{1}{R}\right) = \frac{1}{8R}$
5. This is our most reduced form, so plug the given values into our hand-crafted formula

..continued..

$$P_{pep} = \frac{(2x10^2)^2 {V_{rms}}^2}{(8)(5.0x10^1\Omega)} \tag{6}$$

$$P_{pep} = \frac{(2)^2(10^4)}{(40)(10^1)} W \tag{7}$$

$$P_{pep} = \frac{\cancel{(4)}(10^4)}{\cancel{(4)}(10^2)} W \tag{8}$$

$$P_{pep} = 10^2 W \tag{9}$$

$$\mathbf{P_{pep} = 100W}$$

Notes:

6. Cancel some units which reduce to Watts $\left(\frac{V^2}{\Omega}\right)$ and perform the basic arithmetic as well as some exponent math: $(2x10^2)^2 = (2)^2(10^2)^2$ and that $(10^2)^2 = 10^{(2)(2)} = 10^4$. Recall that we can split $(5.0x10^1)$ into $(5.0)(10^1)$ to make future operations easier to plan out
7. Solve $(2)^2 = 4$ and then convert 40 to $(4x10^1) = (4)(10^1)$ after which we simplify the expression by multiplying the exponentials as: $(10^1)(10^1) = 10^{1+1} = 10^2$
8. Since $\frac{a}{a} = 1$ we can cancel out some values. Also notice that now we have more exponent math: $\frac{10^4}{10^2} = 10^{(4-2)} = 10^2$
9. Technically, we have our answer, however, lets provide it in non-scientific format, in the base unit of Watts

So notice that we obtain an exact value for the peak-envelope-power using this more academic approach? What is exciting is that all the techniques used to manipulate the three equations into a simplified expression (step 5 above) are basic functions and do not require a calculator! In fact, you could memorize:

$$\mathbf{P_{pep} = \frac{(E_{peak-to-peak})^2 (V_{rms})^2}{8R}}$$

Notice how useful an academic approach to solving equations can be very beneficial? We derived the generalized formula for solving peak-envelope-power when using peak-to-peak voltage and a dummy-load resistance. Congrats on reading this far!

Q11: Time Constant: Capacitive
What is the time constant of a circuit having a 100µF capacitor in series with a 470kΩ resistor?

A11: Recall the basic equation of time constant for a capacitor and resistor:

$$\boldsymbol{\tau = RC}$$

given:
$$R = 470\text{k}\Omega = 4\underbrace{70000}\,\Omega = 470 \text{ x } 10^3\Omega = 4.7 \text{ x } 10^5\Omega$$
$$C = 100\mu\text{F} = 100 \text{ x } 10^{-6}\text{F} = 1.0 \text{ x } 10^{-4}\text{F}$$

$$\tau = (4.7 \text{ x } 10^5\Omega)(1.0 \text{ x } 10^{-4}\text{F}) \quad (1)$$

$$\tau = (4.7\Omega)(1.0\text{F})(10^5)(10^{-4}) \quad (2)$$

$$\tau = (4.7\text{s})(10^5)(10^{-4}) \quad (3)$$

$$\tau = (4.7\text{s})(10^1) \quad (4)$$

$$\boldsymbol{\tau = 47s}$$

Notes:

1. Plug the information given to us into the equation. This won't work yet since we are working with numbers with vastly different base units. We need to bring them down to their base SI units. Failing to do so will result in an answer 100s to 1'000'000s too high or too low.
2. Using Commutative properties, we can separate out the scientific notation (eg: 10^5) by rearranging the equation to isolate the variables from their exponential values. Multiply $(4.7\Omega)(1\text{F})$, leaving the exponent math for last. As an aside, it should be noted that $\Omega \text{ x F}$ cancels all dimensions but seconds. See the Appendix for a mathematical explanation.
3. Exponent math: $(10^5)(10^{-4})$ is the same as $10^{(5-4)} = 10^1 = 10$
4. Multiply 4.7s by 10

This equation has very practical uses in creating a timer circuit. As a capacitor charges, it takes 1 time constant for the capacitor's voltage to reach 63.2% of its maximum voltage. Knowing this, if you create a circuit which only "triggers" at a pre-defined voltage, you can vary the time it takes to hit the trigger voltage by modifying either the capacitance, or more commonly, the resistance. Similarly, you can create a circuit that stays on until the discharge voltage is reached at a predetermined time.

Q12: Time Constant: Inductive
What is the time constant of a circuit having a 5mH inductor in series with a 100Ω resistor?

A12: Recall the basic equation of time constant for an inductor and resistor:

$$\boldsymbol{\tau = \frac{L}{R}}$$

given: $L = 5\text{mH} = 5.0x10^{-3}\text{H}$
$R = 100\Omega = 1.0x10^{2}\Omega$

$$\tau = \frac{5.0x10^{-3}\text{H}}{1.0x10^{2}\Omega} \quad (1)$$

$$\tau = \frac{(5.0)(10^{-3}\text{H})}{(1.0)(10^{2}\Omega)} \quad (2)$$

$$\tau = (5)(10^{-5}s) \quad (3)$$

$$\tau = \frac{(50)\cancel{(10^{-6}s)}}{1} x \frac{1\mu s}{\cancel{(10^{-6}s)}} \quad (4)$$

$$\boldsymbol{\tau = 50\mu s}$$

Notes:

1. Input the known values into the equation using scientific notation
2. Separate the coefficient from the exponential value and divide 5 by 1. For the exponent math, notice that $\frac{10^{-3}}{10^{2}} = 10^{(-3)-2} = 10^{-5}$ and also notice that $\frac{H}{\Omega} = s$
3. Since $10^{-5}s$ is an awkward unit, lets convert it to the next nearest metric equivalent which is micro $(\mu) = 10^{-6}$ so to accomplish this, we move the decimal one space to the right, subtracting 1 from the 10^{-5} leaving $(50)(10^{-6}s)$
4. Convert $(50)(10^{-6}s)$ to μs using a conversion fraction and cross-cancel like units

This form of equation is not really useful since the time constant value is generally vanishingly small with appropriately sized values. In this case, 50 microseconds is awfully quick. Note that in inductors, it is the current which reaches 63.2% after 1 time constant. This differs from capacitors in which it is voltage!

Q13: Reactance: Capacitive
What is the capacitive reactance of a 25 microfarad capacitor connected to a 60Hz line?

A13: Recall that capacitors tend to pass more current as you increase the input frequency? So this suggests then that we should be using an equation that looks like a fraction so that as f increases, the value of the reactance decreases. Also recall that virtually any time you need to work with frequency, the term 2πf is often found? Let us therefore use the equation for X_C

$$\boldsymbol{X_C = \frac{1}{2\pi f C}}$$

given: $f = 60Hz$
$C = 25\mu F = 25\text{x}10^{-6}F = 2.5\text{x}10^{-5}F$

$$X_C = (2\pi)^{-1}(fC)^{-1} \quad (1)$$

$$X_C = (2\pi)^{-1}\left((60Hz)(2.5\text{x}10^{-5}F)\right)^{-1} \quad (2)$$

$$X_C = (2\pi)^{-1}\left((6.0)(2.5)(10^{1}Hz)(10^{-5}F)\right)^{-1} \quad (3)$$

$$X_C = (2\pi)^{-1}\left((15)(10^{-4}\Omega^{-1})\right)^{-1} \quad (4)$$

$$X_C = (2\pi)^{-1}(10^{4}\Omega)(15)^{-1} \quad (5)$$

$$X_C = \left(\frac{1}{2\pi}\right)\left(\frac{1}{15}\right)(10^{4}\Omega) \quad (6)$$

$$X_C = \frac{10^{4}\Omega}{30\pi} \quad (7)$$

$$\mathbf{X_C = 106\Omega}$$

Notes:

1. Rearrange the basic equation and then immediately plug all known values into the equation. Be sure you always do this in base units! Failing to do so will lead to errors.
2. Reduce 60Hz to the scientific notation equivalent to handle the exponents
3. Multiply the coefficients and the exponentials as: $(10^{1})(10^{-5}) = 10^{(1-5)} = 10^{-4}$. Notice the new unit: Ω^{-1} (see Appendix for explanation)
4. Now apply the $(\quad)^{-1}$ to everything within the brackets. Recall: $(10^{-4})^{-1} = 10^{(-4)(-1)} = 10^{4}$ and in case you didn't catch it, even the Ω^{-1} was effected, and grouped with the (10^{4})
5. Since $(15)^{-1} = \frac{1}{15}$ use that notation to simplify the math and change $(2\pi)^{-1}$ to its reciprocal form too.
6. Multiply the fractions
7. Solve the division and you are done! Recall that $10^{4} = 1\underbrace{0000}$ or on a calculator, it can be expressed as: $1E4$

Q14: Reactance: Inductive
What is the inductive reactance of a 30 microhenry coil connected to a 120Hz line?

A14: Recall that inductors tend to pass less current as you increase the input frequency? They are the opposite of capacitors in this regard. At zero frequency (ie. DC) the reactance of an ideal coil is zero since an ideal inductor is effectively just a coiled wire with zero resistance!

$$\boldsymbol{X_L = 2\pi f L}$$

given: $f = 120Hz = 1.2x10^2 Hz$
$L = 30\mu H = 30x10^{-6}H = 3.0x10^{-5}H$

$$X_L = (2\pi)(1.2x10^2 Hz)(3.0x10^{-5}H) \quad (1)$$

$$X_L = (2\pi)(1.2)(3.0)(10^2 Hz)(10^{-5}H) \quad (2)$$

$$X_L = (7.2\pi)(10^{-3}\Omega) \quad (3)$$

$$X_L = \frac{(22.6)\cancel{(10^{-3}\Omega)}}{1} x \frac{1m\Omega}{\cancel{(10^{-3}\Omega)}} \quad (4)$$

$$\boldsymbol{X_L = 22.6m\Omega}$$

Notes:

1. Plug the given values into the equation, grouping coefficients and exponentials
2. Multiply all the coefficients, leaving π. Also perform the exponent math as follows: $(10^2)(10^{-5}) = 10^{(2-5)} = 10^{-3}$ and note that $(Hz)(H) = \Omega$
3. Multiply $(7.2)(\pi)$
4. Since we have an awkward result in base units, let's express the value using a metric prefix. Recall that $(10^{-3}\Omega) = 1m\Omega$. Using a conversion fraction, find the answer in $m\Omega$ by cross-cancelling like units

In general, you will not see a value quoted in any unit smaller than Ω unless you are dealing with reactance, or resistor networks which are not common outside of circuit design parameters. For our example, it makes sense that there is very little reactance since the frequency is very low. Try this same example again using 1MHz and you should find your $X_L = 188\Omega$ for the same coil!

Q15: Quality Factor (Q): Parallel Inductor

What is the Q of a parallel R-L-C circuit, if it is resonant at 7.125MHz, L is 10.1µH, and the resistance is 100Ω?

A15: Recall that Q is a dimensionless measure.

$$\boldsymbol{Q_{parallel} = \frac{R}{X_L}}$$

$$\boldsymbol{Q = \frac{R}{2\pi f L}}$$

given: $f = 7.125\text{MHz} = (7.125\text{x}10^6\text{Hz})$
$L = 10.1\mu H = (1.01\text{x}10^{-5}H)$
$R = 100\Omega = (1.0\text{x}10^2\Omega)$

$$Q = \frac{(1.0\text{x}10^2\Omega)}{(2\pi)(7.125\text{x}10^6\text{Hz})(1.01\text{x}10^{-5}\text{H})} \quad (1)$$

$$Q = \frac{(1.0)(10^2\Omega)}{(2\pi)(7.125)(1.01)(10^6 Hz)(10^{-5} H)} \quad (2)$$

$$Q = \frac{(1.0)(10^2\not{\Omega})}{(1.439\pi)(10^1)(10^1\not{\Omega})} \quad (3)$$

$$Q = \frac{(1.0)(\not{10^2})}{(1.44\pi)(\not{10^2})} \quad (4)$$

$$Q = \frac{1}{1.44\pi} \quad (5)$$

$$\mathbf{Q = 0.221}$$

Notes:

1. Input the known values into the equation and separate all coefficients from their exponential values and group similar terms.
2. Multiply all coefficients leaving π intact as: $(2\pi)(7.125)(1.01) = (14.39\pi) = (1.439\pi)(10^1)$. Also perform the exponent math as: $(10^6)(10^{-5}) = 10^{(6-5)} = 10^1$. Notice that $(Hz)(H) = \Omega$
3. Perform the exponent math as: $(10^1)(10^1) = 10^{1+1} = 10^2$ and cancel the units
4. Cancel like terms since $\frac{(10^2)}{(10^2)} = 10^{(2-2)} = 10^0 = 1$
5. Perform the dimensionless (no units here!) arithmetic to solve for Q. Note that on a calculator, it is easy to get the wrong answer unless you work it out as: $1 \div (1.44\ x\ \pi)$

The equation $\boldsymbol{Q_{parallel} = \frac{R}{2\pi f L}}$ is a simplification of a more complex equation: $Q = R\sqrt{\frac{C}{L}}$ and is a little laborious to demonstrate, so I have elected to not fully discuss the derivation of $\boldsymbol{Q_{parallel}}$. On the Advanced exam, you are only asked to find the Q of a parallel R-L-C circuit with induction given. Maybe memorize the simplified $\boldsymbol{Q_{parallel}}$ equation for this reason!

Q16: Resonance: Frequency

What is the resonant frequency of a series R-L-C circuit if R = 47Ω, L = 40µH and C = 300pF?

A16: Ignore the statement "series" as the equation is the same for both series and parallel. Also ignore the resistance since it does not influence resonance (though it does effect Q calculations).

$$\boldsymbol{f} = \frac{\mathbf{1}}{\mathbf{2\pi\sqrt{LC}}}$$

given: $\mathrm{L} = 40\mu\mathrm{H} = 40 \text{ x } 10^{-6}\mathrm{H} = 4.0 \text{ x } 10^{-5}\mathrm{H}$
$\mathrm{C} = 300\mathrm{pF} = 300 \text{ x } 10^{-12}\mathrm{F} = 3.0 \text{ x } 10^{-10}\mathrm{F}$

$$f = \frac{1}{2\pi\sqrt{\mathrm{LC}}} = \left(2\pi\sqrt{\mathrm{LC}}\right)^{-1} = \left(2\pi(\mathrm{LC})^{\frac{1}{2}}\right)^{-1} = (2\pi)^{-1}(\mathrm{LC})^{-\frac{1}{2}} \quad (1)$$

$$f = (2\pi)^{-1}(\mathrm{LC})^{-\frac{1}{2}} \quad (2)$$

$$f = (2\pi)^{-1}\left((4.0 \text{ x } 10^{-5}\mathrm{H})(3.0 \text{ x } 10^{-10}\mathrm{F})\right)^{-\frac{1}{2}} \quad (3)$$

$$f = (2\pi)^{-1}\left((12.0\mathrm{s}^2)(10^{-5})(10^{-10})\right)^{-\frac{1}{2}} \quad (4)$$

$$f = (2\pi)^{-1}\left((1.2\mathrm{s}^2)(10^{1})(10^{-5})(10^{-10})\right)^{-\frac{1}{2}} \quad (5)$$

$$f = (2\pi)^{-1}\left((1.2\mathrm{s}^2)(10^{-14})\right)^{-\frac{1}{2}} \quad (6)$$

Notes:

1. Rearrange the formula to simplify the math later and select the $(2\pi)^{-1}(\mathrm{LC})^{-\frac{1}{2}}$ form for our purposes, which will allow us to perform most calculations without a calculator.
2. Substitute the known L and C values into the equation in their most reduced form
3. Multiply L x C but keep the exponential math separate.
4. Reduce the $12.0\mathrm{s}^2$ to $(1.2\mathrm{s}^2)(10^1)$ and group the (10^1) with the other exponentials. Notice the unit s^2 (see Appendix).
5. Perform the exponent math as follows: $(10^1)(10^{-5})(10^{-10}) = \left(10^{(1-5-10)}\right) = 10^{-14}$
6. Apply the $(\quad)^{-\frac{1}{2}}$ to the term $(1.2\mathrm{s}^2)$ making it $(1.2\mathrm{s}^2)^{-\frac{1}{2}}$ and then do the same thing to the (10^{-14}) term, making it $(10^{-14})^{-\frac{1}{2}}$

.. continued..

$$f = (2\pi)^{-1}(1.2s^2)^{-\frac{1}{2}}(10^{-14})^{-\frac{1}{2}} \quad (7)$$

$$f = (10^7)\left(\frac{1}{\sqrt{1.2s^2}}\right)\left(\frac{1}{2\pi}\right) \quad (8)$$

$$f = \frac{10^7}{2\pi\sqrt{1.2s^2}} \quad (9)$$

$$f = \frac{10^7}{(2.191s)\pi} \quad (10)$$

$$f = 1.452 \text{ x } 10^6 \text{ s}^{-1} \quad (11)$$

$$f = \frac{1.45 \text{ x } \cancel{10^6\text{Hz}}}{1} \text{ x } \frac{1\text{MHz}}{\cancel{10^6\text{Hz}}} \quad (12)$$

$$\mathbf{f = 1.45MHz}$$

Notes:

7. Solve the $(10^{-14})^{-\frac{1}{2}}$ as $10^{(-14)\left(-\frac{1}{2}\right)} = 10^{\frac{14}{2}} = 10^7$ and show the $(1.2s^2)^{-\frac{1}{2}}$ term as $\frac{1}{\sqrt{1.2s^2}}$ and $(2\pi)^{-1}$ as $\frac{1}{2\pi}$ to simplify the math later
8. Simplify the equation since: $\left(\frac{1}{\sqrt{1.2s^2}}\right)\left(\frac{1}{2\pi}\right) = \frac{1}{2\pi\sqrt{1.2s^2}}$
9. Solve $2\sqrt{1.2s^2}$ as $2.191s$ (rounded since we are limited by significant figures) and leave the π alone for now. Notice that $\sqrt{s^2} = (s^2)^{\frac{1}{2}} = s^{\left(\frac{2}{1}\right)\left(\frac{1}{2}\right)} = s^{\frac{2}{2}} = s^1 = s$. This is merely an academic point, it is not critical to remember. Just know it reduces to seconds
10. Solve the equation to express the answer in reciprocal seconds (s^{-1}) *or Hz*! To do this on your calculator, you can solve it as: $1E7 \div (2.191\pi)$ (see primer for 1E7).
11. Dividing $\frac{1.591}{1.095s}$ we find it is $\frac{1.452}{s} = 1.452s^{-1}$ strange, we are left with reciprocal seconds. See Appendix for an explanation.
12. Convert the $1.452s^{-1}$ to Hz since s^{-1} is the same as Hz. Now Convert the Hz to MHz to get rid of the 10^6. Do this by multiplying by $\frac{1\text{MHz}}{10^6\text{Hz}}$ which effectively means $1\text{MHz} = 10^6\text{Hz}$

On the advanced exam, there are always two questions asking you to find the resonant frequency of an R-L-C circuit. It behoves you to become very familiar with this process if you wish to do well on the advanced exam.

Q17: Resonance: Solving for C
What is the value of capacitance (C) in a series L-R-C circuit, if the circuit resonant frequency is 14.25MHz and L is 2.84 microhenrys?

A17: So we know the basic formula for finding resonance, thus, we simply rearrange the equation to solve for the missing value (C). This is the longer (and preferred) method, however, on an exam you can simply substitute the most appropriate value(s) and test by seeing if your resonant frequency is the same as the given value.

$$\mathbf{f = \frac{1}{2\pi\sqrt{LC}}}$$

given: $L = 2.84\mu H = 2.84\text{x}10^{-6}H$
$f = 14.25MHz = 1.425\text{x}10^{7}Hz$

$$f = \left(2\pi\sqrt{LC}\right)^{-1} \quad (1)$$

$$f^{-2} = \left(\left(2\pi\sqrt{LC}\right)^{-1}\right)^{-2} \quad (2)$$

$$f^{-2} = \left(2\pi\sqrt{LC}\right)^{2} \quad (3)$$

$$f^{-2} = (2\pi)^2 LC \quad (4)$$

$$\frac{f^{-2}}{(2\pi)^2 L} = \frac{(2\pi)^2 LC}{(2\pi)^2 L} \quad (5)$$

$$C = \frac{\frac{1}{f^2}}{(2\pi)^2 L} \quad (6)$$

Notes:

1. First, make the equation linear by expressing the right side of the equation as an inverse $a^{-1} = \frac{1}{a}$
2. To rearrange the equation in order to isolate C, we must start by getting rid of the $\sqrt{\ }$ since the value we seek is found within the $\sqrt{\text{function}}$. How do we get rid of a $\sqrt{\ }$? Simply square both sides of the equation since: $\left(\sqrt{a}\right)^2 = \left(a^{\frac{1}{2}}\right)^2 = a^{\left(\frac{1}{2}\right)(2)} = a^{\frac{2}{2}} = a^1 = a$ and recall that the exponent math goes as: $\left(a^b\right)^c = a^{(b)(c)}$. But wait! Let's take the negative square root!
3. We chose the negative square to make it easier to manipulate the $\left(2\pi\sqrt{LC}\right)^2$ which we will now solve as: $(2\pi)^2\left(\sqrt{LC}\right)^2 = (2\pi)^2 LC$.
4. Divide both sides of the equation by $(2\pi)^2 L$ to isolate C, cancelling where possible due to the relationship between $\frac{a}{a} = 1$
5. Recall that $a^{-2} = \frac{1}{a^2}$ which we use to express f^{-2} now
6. Ugly huh? Solve $(2\pi)^2$ as $(2)^2(\pi)^2 = 4\pi^2$ and the fraction as: $\frac{\frac{1}{a}}{b} = \frac{\frac{1}{a}}{\frac{b}{1}} = \left(\frac{1}{a}\right)\left(\frac{1}{b}\right) = \frac{1}{ab}$

..continued..

$$\mathbf{C = \frac{1}{4\pi^2 Lf^2}} \quad (7)$$

At this point, we have manipulated the original $f = \frac{1}{2\pi\sqrt{LC}}$ equation to a point where we can solve for the capacitance required to achieve resonance when a resonator of inductance L is used. It should be noted that C and L can be interchanged in equation 7 since they were both subject to the $\sqrt{\ }$

$$C = (4\pi^2 Lf^2)^{-1} \quad (8)$$

$$C = (4\pi^2(2.84)(H)(10^{-6})(1.425)^2(Hz)^2(10^7)^2)^{-1} \quad (9)$$

$$C = \left((\pi^2)(4)(2.84)(\Omega s)(2.030)\left(\frac{1}{s^2}\right)(10^{-6})(10^{14})\right)^{-1} \quad (10)$$

$$C = \left((\pi^2)(2.306)\left(\frac{\Omega}{s}\right)(10^1)(10^8)\right)^{-1} \quad (11)$$

$$C = \left((\pi^2)\left(2.306\left(\frac{\Omega}{s}\right)\right)(10^9)\right)^{-1} \quad (12)$$

$$C = (\pi^{-2})(2.306)^{-1}\left(\frac{s}{\Omega}\right)(10^{-9}) \quad (13)$$

Notes:

7. And here we are, the completed formula. Notice that we leave π^2 alone? This gives greater accuracy later, and is generally the preferred method.
8. Rearrange the reciprocal function to an exponential form for easier exponent math later. Plug our known values into the finished equation and then reduce / group terms where possible.
9. Solve $(1.425\text{Hz})^2$ and $(10^7)^2 = 10^{(7)(2)}$ and put the units in where $H = \Omega s$ and $Hz = \frac{1}{s^2}$
10. Work out $(10^{-6})(10^{14}) = 10^{-6+14} = 10^8$ and $(4)(2.84\Omega s)\left(2.030\frac{1}{s^2}\right) = \left(2.306\left(\frac{\Omega}{s}\right)\right)(10^1)$ noting how the units reduce: $\left(\frac{\Omega s}{1} x \frac{1}{s^2}\right) = \left(\frac{\Omega}{s}\right)$
11. Simplify $(10^1)(10^8) = (10^9)$
12. Apply the $(\quad)^{-1}$ to all terms, and notice that $\left(\frac{\Omega}{s}\right)$ becomes $\left(\frac{s}{\Omega}\right)$ which is the units for Farads!
13. Since $(\pi^{-2})(2.306)^{-1} = \frac{1}{(2.306)(\pi^2)}$ we will use this fact to clean up our equation. We will also substitute in F for $\left(\frac{s}{\Omega}\right)$

..continued..

$$C = \frac{(10^{-9})(F)}{(2.306)(\pi^2)} \quad (14)$$

$$C = \frac{(10^{-9})(F)}{(2.275)(10^1)} \quad (15)$$

$$C = \frac{(10^{-10}F)}{(2.275)} \quad (16)$$

$$C = \frac{1}{2.275} x(10^{-10}F) \quad (17)$$

$$C = (0.4395)(10^{-10}F) \quad (18)$$

$$C = (44.0)(10^{-2})(10^{-10}F) \quad (19)$$

$$C = (44.0)\frac{(\cancel{10^{-12}F})}{1} x \left(\frac{1pF}{\cancel{10^{-12}F}}\right) \quad (20)$$

$$\mathbf{C = 44.0pF}$$

Notes:

14. Finally reduce $(2.306)(\pi^2) = (2.275)(10^1)$
15. Solve some exponent math $\frac{10^{-9}}{10^1} = 10^{(-9)-(1)} = 10^{-10}$
16. Express the equation in a more intuitive format
17. Solve the equation, expressing the answer in farads
18. Re-express the answer in scientific notation. Notice that we elected to use $(43.9x10^{-2})$ as opposed to $(4.39x10^{-3})$, this is a result of some intuition as to how we can later express the answer. It is also an appropriate time to round our answer down to 3 decimal places due to our limited accuracy due to so-called "sig-figs" (significant digits/figures).
19. Multiply the exponentials as $(10^{-2})(10^{-10}F) = (10^{-12}F)$ which looks familiar, doesn't it? It should since $1pF = 10^{-12}F$
20. Use one of our trusty unit conversion fractions to convert the $(10^{-12}F)$ to pF

Ok so that one was a little bit long, but it was important to demonstrate how you can manipulate an equation to solve for a variable of interest - even when it seems that the variable you want is buried away. On the exam, remember that it may be quicker just to "plug and pray" (ie. try the given answers as values in the resonant frequency equation to see which one gives you the same resonant frequency as the question provides.)

Q18: Transformer Ratio: Voltage and Current

A transformer has a primary voltage of 240V and draws a current of 250mA from the wall outlet. Assuming no losses, what is the maximum current available from the secondary which outputs 12V?

A18: Recall that transformers can increase or decrease voltage but they do so at the expense of current. In this case, the transformer decreases the available voltage but this means it must therefore increase the available current (according to the law of conservation of energy). So the question asks us to find I_S

$$\mathbf{\frac{N_P}{N_S} = \frac{E_P}{E_S} = \frac{I_S}{I_P}}$$

given: $E_P = 240V$ $\quad I_P = 250mA = 2.5x10^{-1}A$

$E_S = 12V$

$$\frac{E_P}{E_S} = \frac{I_S}{I_P} \quad (1)$$

$$I_S = \frac{(E_P)(I_P)}{(E_S)} \quad (2)$$

$$I_S = \frac{(240V)(2.5x10^{-1}A)}{(12V)} \quad (3)$$

$$I_S = \frac{(2.4)(2.5A)(10^2)(10^{-1})}{(1.2)(10^1)} \quad (4)$$

$$I_S = \frac{6A}{1.2} \quad (5)$$

$$\mathbf{I_S = 5A}$$

Notes:

1. First, we use the "transformer turns" ratio selecting for the variables we are given: E and I. Notice that unlike N_p and E_p in the numerator, the value I_p is in the denominator. This is very important! Rearrange the equation, isolating I_S since that is what we need to calculate. To do so, multiply both sides of the equation by I_p
2. Input the given values into the equation and cancel any like terms (V)
3. Express all values in scientific notation and separate coefficients from their exponentials to simplify the math
4. Multiply all like terms and perform the exponent math $\frac{(10^2)(10^{-1})}{(10^1)} = \frac{10^{(2-1)}}{10^1} = \frac{10^1}{10^1} = 1$
5. Solve the final division

Q19: Wavelength: Free-space
What is the wavelength of a radio signal sent at a frequency of 4.0MHz

A19: Using the relationship between frequency, wavelength and the speed of light, we can find the wavelength (in free space) of a given frequency (and vice versa). The constant "'c"' is the speed of light in a vacuum, and is said to be approximated as $3.0\text{x}10^8\text{ms}^{-1}$ and therefore should be committed to memory.

$\boldsymbol{\lambda = \frac{c}{f}}$

$\lambda = \text{wavelength (m)}$

$\text{c} = \text{the speed of light} \left(\frac{\text{m}}{\text{s}}\right) \text{ or } (\text{ms}^{-1}) = (3.0\text{x}10^8\text{ms}^{-1})$

$f = \text{frequency in (Hz) or } (\text{s}^{-1})$

given: $f = 4.0\text{MHz} = (4.0\text{x}10^6\text{s}^{-1})$

$$\lambda = \frac{(3\text{x}10^8\text{ms}^{-1})}{(4.0\text{x}10^6\text{s}^{-1})} \quad (1)$$

$$\lambda = \frac{(3\text{x}10^8\text{m})}{(4.0\text{x}10^6)} \quad (2)$$

$$\lambda = \frac{(3\text{m})(10^8)}{(4.0)(10^6)} \quad (3)$$

$$\lambda = \frac{(3\text{m})(10^2)}{(4.0)} \quad (4)$$

$$\lambda = \frac{(300\text{m})}{(4.0)} \quad (5)$$

$\boldsymbol{\lambda = 75\text{m}}$

Notes:

1. Substitute the known values into the base equation and cancel like terms
2. Separate coefficients from the exponentials
3. Perform the exponential math as follows: $\frac{(10^8)}{(10^6)} = 10^{(8-6)} = 10^2$
4. For the sake of ease, solve $(3\text{m})(10^2)$
5. Divide 300m by 4.0 to calculate the wavelength

This equation is only valid for wavelengths in free space (ie. a vacuum). For antenna length, the following equation is generally widely used which gives the $^1/_2\,\lambda$ value in feet (ick):

$$\boldsymbol{^1/_2\,\lambda\ \text{dipole} = \frac{468\text{ft}}{f\ (\text{MHz})}}$$

Q20: Wavelength: Practical Antennas
What would be the physical length of a typical coaxial stub that is electrically one quarter wavelength long at 14.1MHz (assuming a velocity factor of 0.66)?

A20: This is a two-step process which we will combine into a single equation. First, we need to calculate the free-space wavelength of 14.1MHz, then we will use that information to solve the electrical one quarter wavelength in coaxial cable.

$$\boldsymbol{\lambda_{\text{free-space}} = \frac{c}{f}\ \lambda = \text{wavelength (m)}}$$

$$\boldsymbol{\frac{1}{4}\lambda_{\text{electrical}} = \frac{(\lambda_{\text{free-space}})(v_p)}{4}} \qquad v_p = \text{velocity of propagation (velocity factor)}$$

given: $c = 3.0\text{x}10^8\text{ms}^{-1}$
$f = 14.1\text{MHz} = 1.41\text{x}10^7\text{s}^{-1}$
$v_p = 0.66$

$$\frac{1}{4}\lambda_{\text{electrical}} = \frac{\left(\frac{c}{f}\right)(v_p)}{4} \quad (1)$$

$$\frac{1}{4}\lambda_{\text{electrical}} = \frac{(c)(v_p)}{4f} \quad (2)$$

$$\frac{1}{4}\lambda_{\text{electrical}} = \frac{(3.0\text{x}10^8\text{ms}^{-1})(0.66)}{(4)(1.41\text{x}10^7\text{s}^{-1})} \quad (3)$$

$$\frac{1}{4}\lambda_{\text{electrical}} = \frac{(3.0\text{m})(0.66)(10^8)}{(4)(1.41)(10^7)} \quad (4)$$

$$\frac{1}{4}\lambda_{\text{electrical}} = \frac{(1.98\text{m})(10^1)}{(5.64)} \quad (5)$$

$$\frac{1}{4}\lambda_{\text{electrical}} = \frac{(19.8\text{m})}{(5.64)} \quad (6)$$

$$\boldsymbol{\frac{1}{4}\lambda_{\text{electrical}} = 3.51\text{m}}$$

Notes:

1. Take the electrical wavelength equation and plug the free-space equation into it. Simplify the equation such that $\frac{\left(\frac{a}{b}\right)(c)}{d} = \frac{\frac{ac}{b}}{d} = \frac{\frac{ac}{b}}{\frac{d}{1}} = \left(\frac{ac}{b}\right)\left(\frac{1}{d}\right) = \frac{ac}{bd}$
2. Plug the known values into the simplified equation.
3. Group coefficients and the exponentials and cancel units where possible.
4. Perform the arithmetic on the coefficients and the exponentials $\frac{(10^8)}{(10^7)} = 10^{(8-7)} = 10^1$
5. Simplify the numerator
6. Solve the division

Q21: Multimeters: Increasing effective range

A voltmeter having a range of 150 volts and an internal resistance of 150kΩ is to be extended to read up to 750 volts. The required multiplier resistor would have what value?

A21: First we need to note that we wish to extend the voltage scale which means we would add a resistor in series! Thus, the additional resistance is in addition to the existing 150kΩ. Next we solve by setting all values in the form of a ratio and solve for the unknown resistance.

$$\frac{\boldsymbol{V_{nominal}}}{\boldsymbol{V_{extended}}} = \frac{\boldsymbol{R_{internal}}}{\boldsymbol{R_{total}}}$$

$$\boldsymbol{R_{total} = R_{internal} + R_{additional}}$$

given: $V_{nominal} = 150V$
$R_{internal} = 150k\Omega = 1.5x10^4\Omega$
$V_{extended} = 750V$

$$R_{total} = \frac{(R_{internal})(V_{extended})}{V_{nominal}} \quad (1)$$

$$R_{total} = \frac{(1.5x10^4\Omega)(750V)}{150V} \quad (2)$$

$$R_{total} = \frac{(\not{1.5})(10^4\Omega)(750)}{(\not{1.5})(10^1)} \quad (3)$$

$$R_{total} = (10^3\Omega)(750) \quad (4)$$

$$\boldsymbol{R_{total} = 750k\Omega}$$

$$R_{additional} = R_{total} - R_{internal} \quad (5)$$

$$R_{additional} = 750k\Omega - 150k\Omega \quad (6)$$

$$\boldsymbol{R_{additional} = 600k\Omega}$$

Notes:

1. Rearrange the original ratio (equation) to isolate R_{total}
2. Plug the given values into the equation
3. Cancel like terms (1.5) and perform the exponent math: $\frac{10^4}{10^1} = 10^{(4-1)} = 10^3$
4. Recall that $10^3\Omega = 1k\Omega$ so express the intermediate answer as kΩ
5. Rearrange the R_{total} equation to isolate $R_{additional}$ which is what we are looking for
6. Plug the values into the equation and solve the basic arithmetic

So as we have found, it is possible to extend the useful range of a multimeter given we know the internal resistance of the unit. We simply solve a ratio to find the total resistance necessary for a greater voltage scale, and then subtract the known internal resistance from the derived value to determine the value of a series resistor to be used. Just be sure to select a resistor with the appropriate power handling!

Discussion: Advanced Exam Topics

1. Time constants, Back EMF.

The very first question on the advanced exam deals with the concept of time constants. It is imperative to know that in capacitors, one time constant represents the time it takes for the capacitor's voltage to reach 63.2% of the maximum. In an inductor, it is the time required for the current to reach 63.2% of the maximum. It is not uncommon to encounter a question which asks the time it takes for a capacitor to discharge to 36.8% of the starting voltage.

You may be asked to solve the value of time constant for a resistor-capacitor circuit, so be prepared to multiply the resistance by capacitance!

On occasion, you may be asked a question about back EMF which is simply a voltage which opposes an applied voltage. This is analogous to the concept of a reactionary force in classical dynamics.

2. Skin effect, Electrostatic Field, Electromagnetic Field.

Skin effect describes how RF energy flows along the surface of a conductor. It should be noted that as frequency increases, the energy penetration of the current decreases (flowing further away from the center of the conductor).

An electrostatic field is essentially the force of electrical attraction which exists in a capacitor. The unit of measure for capacitance is Farads. Thus,

Appendix:

Converting units:

$$1\text{M}\square = 10^{6};\ 1\text{k}\square = 10^{3};\ 1\square = 10^{0};\ 1\text{m}\square = 10^{-3};\ 1\mu\square = 10^{-6};\ 1\text{n}\square = 10^{-9};\ 1\text{p}\square = 10^{-12}$$

note: (µ = micro)

Recall that these are the metric prefixes. They are an important concept that you must know well! If in doubt, just remember the order from largest to smallest, knowing that in scientific notation, the 10^{x} decreases by 3 for each step. Eg: if you wish to convert from 1M to 1k, the 10^{x} goes from 10^{6} to 10^{3}

To convert from any unit to another, you can use multiplication, provided you know how many of one unit are in the other.

Eg: $0.000041\text{kA} = \text{how many nA?}$

$1\text{kA} = 10^{3}\text{A}$
$1\text{nA} = 10^{-9}\text{A}$

Start by converting each value into a common base unit (A):

$$\frac{0.000041\text{kA}}{1}\text{x}\frac{10^3\text{A}}{1\text{kA}} = \left(0.\underbrace{00004}1\right)(10^3\text{A}) = (4.1\text{x}10^{-5})(10^3\text{A}) = 4.1\text{x}10^{-2}\text{A}$$

$1\text{nA} = 10^{-9}\text{A}$

now we can multiply the $4.1\text{x}10^{-2}\text{A}$ by $1\text{nA} = 10^{-9}\text{A}$

$$\frac{(4.1)(10^{-2}\text{A})}{1} = \frac{1\text{nA}}{(10^{-9}\text{A})} = \frac{(4.1)(1\text{nA})}{10^{-7}} = (4.1\text{nA})(10^{-7})^{-1} = (4.1\text{nA})(10^{7}) = 4.1\text{x}10^{7}\text{nA}$$

$4.1\text{x}10^{7}\text{nA}$ is not a practical unit, however, now you know how to convert to any unit

Oh, and it should be noted that because $1\text{kA} = 10^3\text{A}$, the reverse can also be said:

$1\text{A} = 10^{-3}\text{kA}$

Just note the change in the prefixes and therefore the exponent's sign (positive to negative).

Basic units and their derivations:

$\Omega = \frac{\text{V}}{\text{A}}$ and so $\Omega^{-1} = \frac{\text{A}}{\text{V}}$ $\qquad$ $\text{V} = \frac{\text{W}}{\text{A}}$

To prove that Ohms times Farads equals seconds - $\Omega x F = s$:

$\Omega = \frac{\text{V}}{\text{A}}$ (by Ohms law) and $\text{F} = \frac{(\text{A})(\text{s})}{\text{V}}$

so $\Omega \text{ x F} = \frac{\text{V}}{\text{A}}\text{x}\frac{(\text{A})(\text{s})}{\text{V}} = \frac{\text{s}}{1} = \text{s}$

or more simply:

$\text{F} = \frac{\text{s}}{\Omega}$ and so as above,

$\Omega \text{ x F} = \frac{\Omega}{1}\text{x}\frac{\text{s}}{\Omega} = \frac{\text{s}}{1} = \text{s}$

To prove that Henrys times Farads equals Hertz (Hz) - $HxF = Hz$:

Hertz is cycles per second, a reciprocal expressed as $\frac{\text{cycles}}{\text{s}}$ which can be expressed as s^{-1}

$\text{H} = (\Omega)(\text{s})$ and $\text{F} = \frac{\text{s}}{\Omega}$ we will now use the resonant frequency equation: $f = \frac{1}{2\pi\sqrt{LC}}$

Let's focus on the term (*LC*) in the context of the equation, and recall that we use base-units in our equations, thus *L*=H and *C*=F:

$LC = (\Omega)(\mathrm{s}) \times \frac{\mathrm{s}}{\Omega} = \frac{(\Omega)(\mathrm{s})(\mathrm{s})}{\Omega} = \frac{\mathrm{s}^2}{1} = \mathrm{s}^2$

Since π is a constant, we can ignore it with the rest of the integer values, thus the simplified resonant frequency equation (for the purposes of determining units of measure) is:
$f = \frac{1}{\sqrt{LC}}$
Substituting in our derived value of LC = s^2 we find:
$(LC)^{-\frac{1}{2}} = (\mathrm{s}^2)^{-\frac{1}{2}} = \mathrm{s}^{-1}$

To prove that Henrys times Hz equals Ohms - $(\Omega)(s)(s^{-1}) = \Omega$:

$\mathrm{Hz} = (\mathrm{s})^{-1} = \left(\frac{1}{s}\right)$
$\mathrm{H} = (\Omega)(\mathrm{s})$

so $(\mathrm{Hz})(\mathrm{H}) = (\Omega)(\mathrm{s})\left(\frac{1}{\mathrm{s}}\right) = \frac{(\Omega)(\mathrm{s})}{(\mathrm{s})} = \Omega$

To prove that $Volts^2$ divided by Ohms equals Watts - $\left(\frac{V^2}{\Omega}\right) = W$:

$P = \frac{E^2}{R} = \frac{V^2}{\Omega}$ and power (P) is expressed in watts.

a more rigorous examination using amps goes as follows:

$E = IR$ and

$P = \frac{E^2}{R}$

so $P = \frac{(IR)^2}{R} = I^2R$

$I = A = \frac{C}{s}$

$R = \Omega = \frac{Js}{C^2}$

Thus $P = \left(\frac{C}{s}\right)^2\left(\frac{Js}{C^2}\right) = \left(\frac{\cancel{C^2}}{\cancel{s^2}}\right)\left(\frac{J\cancel{s}}{\cancel{C^2}}\right) = \frac{J}{s}$

And by Classical mechanics, we know a watt to be a measure of joules per second.